KINEMATICS – Test #2

SCAN 70-71-72

Four directional wheel system

Figure 1 represents a part of a mechanical system used for imposing a simultaneous orientation of the front and rear wheels of a vehicle (Honda system).

The mechanism is planar (*within the plane* $(O_0, \vec{x}_0, \vec{z}_0)$), it is formed of 5 solids, namely:

- solid 0: reference, inner ring of centre O_0 and radius $2R$, the dimensions are such that $O_0\vec{D} = -H.\vec{x}_0 + L.\vec{z}_0$
- solid 1: crank of length $O_0A = R$, manivelle
- solid 2: satellite (disc) of centre A and radius R, a pin B is located at a distance $R/2$ from A, doigt
- solid 3 is a bar,
- solid 4 is formed of the female part of the prismatic joint between 4 and 3 plus a bar (*output of the mechanism*).

Parameters:

1/0: revolute joint of axis $(O_{0,1}, \vec{y}_{0,1})$, parameter ψ_1

2/1: revolute joint of axis $(A, \vec{y}_{1,2})$, parameter θ_2

4/0: prismatic joint of axis $(D, \vec{z}_{0,4})$, parameter $Z = D\vec{C}.\vec{z}_{0,4}$

3/4: prismatic joint of axis $(B, \vec{x}_{3,4})$, parameter $X = C\vec{B}.\vec{x}_{3,4}$

3/2: revolute joint of axis $(B, \vec{y}_{3,2})$, ***no parameter.***

2/0: point contact in I, ***it is assumed that there is no slip in I.***

Questions:

1 – Frame definition. Change of basis diagrams. Graph of links.

2 – Constraint equation(s).

a) assuming that θ_2 and ψ_1 are nil at the initial time, prove that $\theta_2 = -2\psi_1$

b) deduce the relationships between Z and ψ_1, between X and ψ_1.

3 – Motion of 3 / 0 :

a) nature, trajectory of B.

4 – Motion of 3 / 4 :

a) nature, trajectory of B.

b) Determine $\vec{V}_3^4(C)$.

5 – Define the axodes of the motion 2 / 0.

6 – $\vec{V}_1^0(A)$ being given, the objective is to determine $\vec{V}_4^0(C)$ graphically (*use appendix I*);

a- Nature of 2/0? Deduce $\vec{V}_{2,3}^0(B)$

b- From the properties of 3/0, draw $\vec{V}_3^0(C)$

c- Using the combination of velocities for ¾, 3/0 and 4/0, find $\vec{V}_4^0(C)$

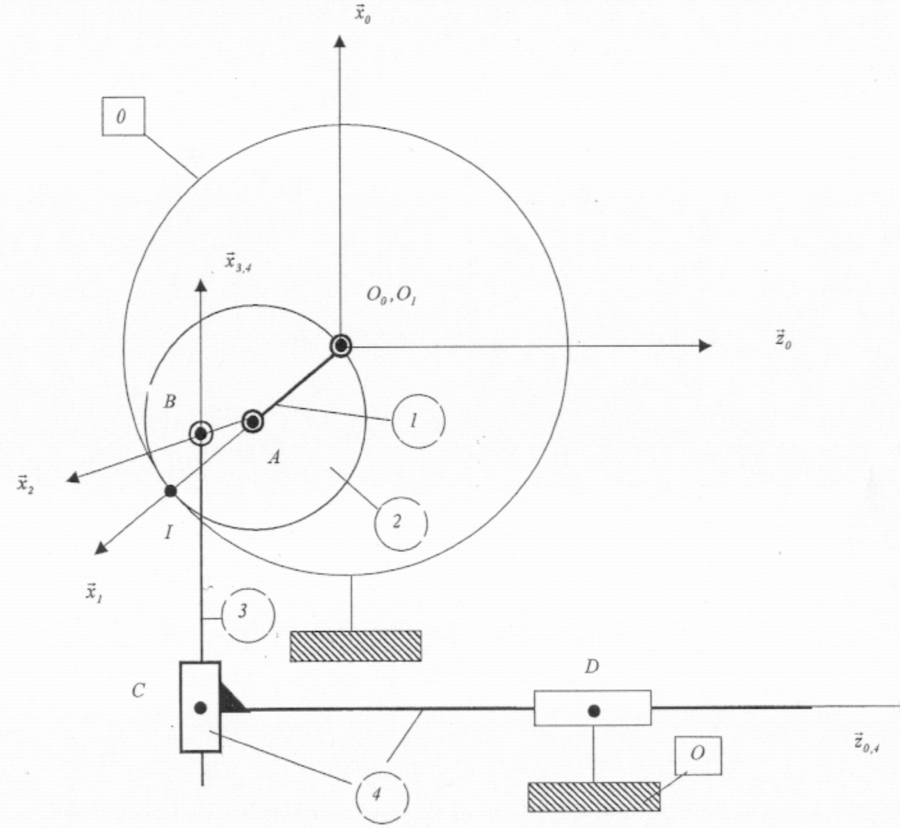
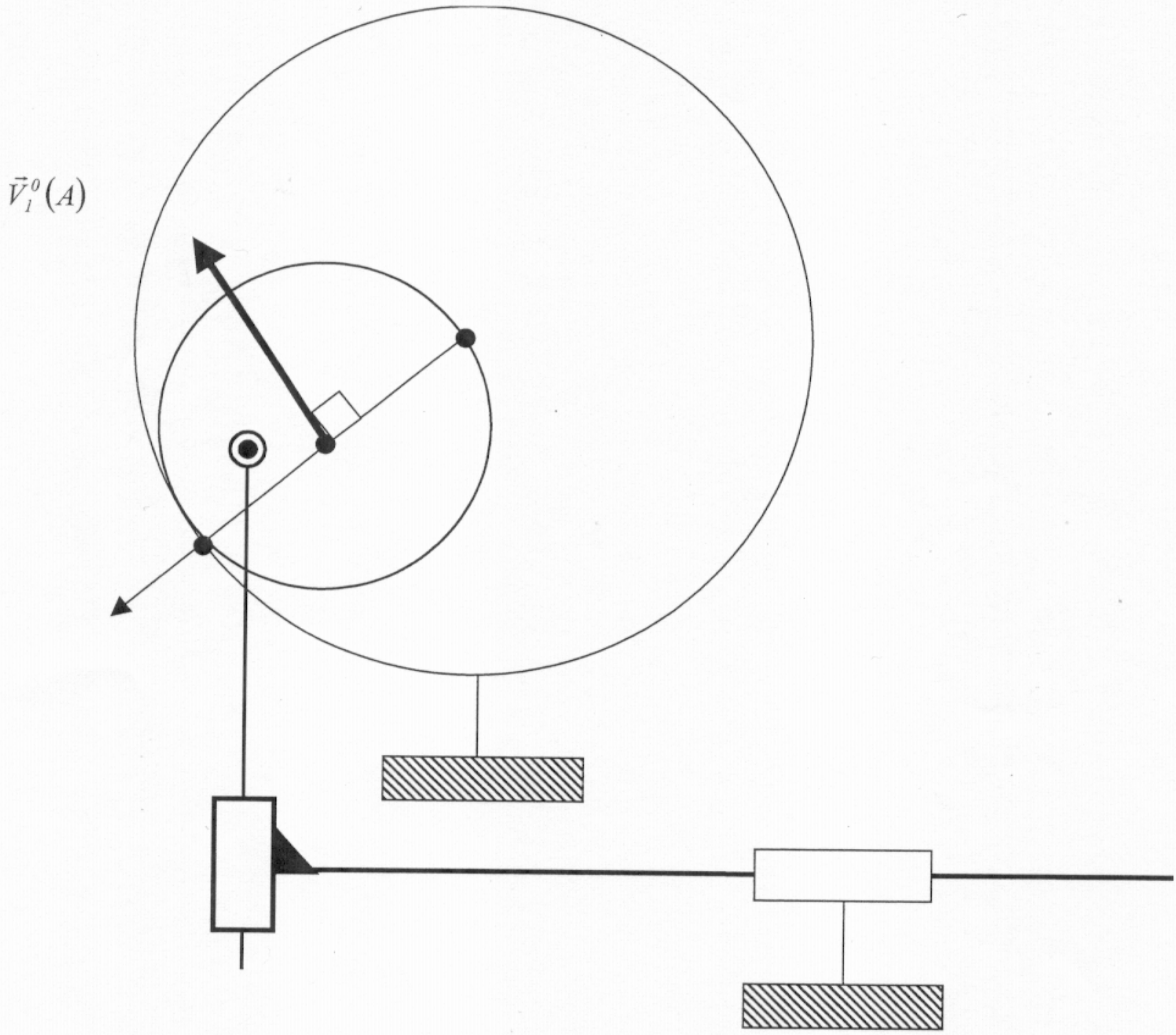


Figure 1 – Schematics of the mechanical system

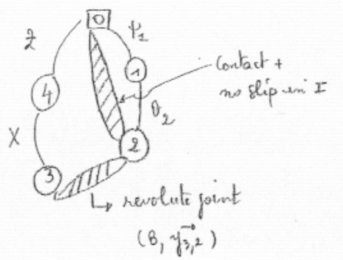
Name:
Group:



Appendix 1 – Graphical kinematics

Kinematics - Test # 2
Elements of correction

1.



2- No slip in I, I fixed in (R_1)

$\vec{V}_2^0(I) = \vec{0}$; $\vec{V}_2^1(I) + \vec{V}_1^0(I) = \vec{0}$; $\cancel{\vec{V}_2^1(A)} + \vec{\Omega}_2^1 \wedge \vec{AI} + \cancel{\vec{V}_1^0(O_1)} + \vec{\Omega}_1^0 \wedge \vec{O_1 I} = \vec{0}$

$\dot{\theta}_2 \vec{y} \wedge R\vec{x}_1 + \dot{\psi}_1 \vec{y} \wedge 2R\vec{x}_1 = \vec{0}$; $-R\left[\dot{\theta}_2 + 2\dot{\psi}_1\right]\vec{z}_1 = \vec{0}$; $\perp \underset{\vec{n}_I}{\overset{\parallel}{\vec{x}_1}} \Rightarrow$ ok

$\Rightarrow$ $\boxed{\dot{\theta}_2 + 2\dot{\psi}_1 = 0}$ ... (1) → integration (initial conditions) } → $\boxed{\theta_2 = -2\psi_1}$ ... (1^*)

Revolute joint in B between 3 and 2

$\begin{cases} \vec{B_2 B_3} = \vec{0} \\ \vec{y}_3 \parallel \vec{y}_2 \end{cases}$ → gives nothing (planar system)

$\vec{B_2 A} + \vec{AO_0} + \vec{O_0 D} + \vec{DC} + \vec{CB_3} = \vec{0}$

$-\frac{R}{2}\vec{x}_2 - R\vec{x}_1 + L\vec{z}_0 - H\vec{x}_0 + Z\vec{z}_{0,4} + X\vec{x}_{3,4} = \vec{0}$

$/\vec{x}_0$: $\boxed{-\frac{R}{2}\cos(\psi_1+\theta_2) - R\cos\psi_1 - H + X = 0}$ --- (2)

$/\vec{z}_0$: $\boxed{\frac{R}{2}\sin(\psi_1+\theta_2) + R\sin\psi_1 + L + Z = 0}$ --- (3)

using (1^*) → $-\frac{R}{2}\cos\psi_1 - R\cos\psi_1 - H + X = 0$ → $X = H + \frac{3}{2}R\cos\psi_1$

→ $-\frac{R}{2}\sin\psi_1 + R\sin\psi_1 + L + Z = 0$ → $Z = -L - \frac{R}{2}\sin\psi_1$

3- Motion 3/0

$\vec{x}_3 \parallel \vec{x}_{4,0}$ $\quad \vec{z}_4 \parallel \vec{z}_0$ → $\begin{cases} \vec{z}_3 \parallel \vec{z}_0 \\ \vec{x}_3 \parallel \vec{x}_0 \\ \vec{y}_3 \parallel \vec{y}_0 \end{cases}$ (planar → 3/0 is a t

(OR) $\vec{\Omega}_3^0 = \vec{\Omega}_3^4 + \vec{\Omega}_4^0 = \vec{0} + \vec{0}$ → same conclusion

traj. of B / (R_0) : $\vec{O_0 B} = \begin{bmatrix} R\cos\psi_1 + \frac{R}{2}\cos(\psi_1+\theta_2) \\ 0 \\ -R\sin\psi_1 - \frac{R}{2}\sin(\psi_1+\theta_2) \end{bmatrix}_{/0}$ (or $\vec{DB} = \begin{bmatrix} X \\ 0 \\ Z \end{bmatrix}$

→ $\begin{cases} x_B = \frac{3R}{2}\cos\psi_1 \\ z_B = -\frac{R}{2}\sin\psi_1 \end{cases}$ → $\frac{x_B^2}{(3R/2)^2} + \frac{z_B^2}{(R/2)^2} = 1$ → ellipse

4- Motion 3/4

. 3/4 is a translation

. traj. of B / (R_4) $\quad \vec{CB} = X\vec{x}_4$ → segment on $(C, \vec{x}_{3,4})$

. $\vec{V}_3^4(C) = \frac{d^4}{dt}\vec{CB} = \frac{d^4}{dt} X\vec{x}_{3,4} = \dot{X}\vec{x}_{3,4}$

5- Axodes 2/0

central axis $(I, \vec{y})$

Axodes $\begin{cases} (\Sigma_2) = \text{cylinder of centre A, radius R} \\ (\Sigma_0) = \text{cylinder of centre } O_0\text{, radius 2R} \end{cases}$

6- Graphical kinematics

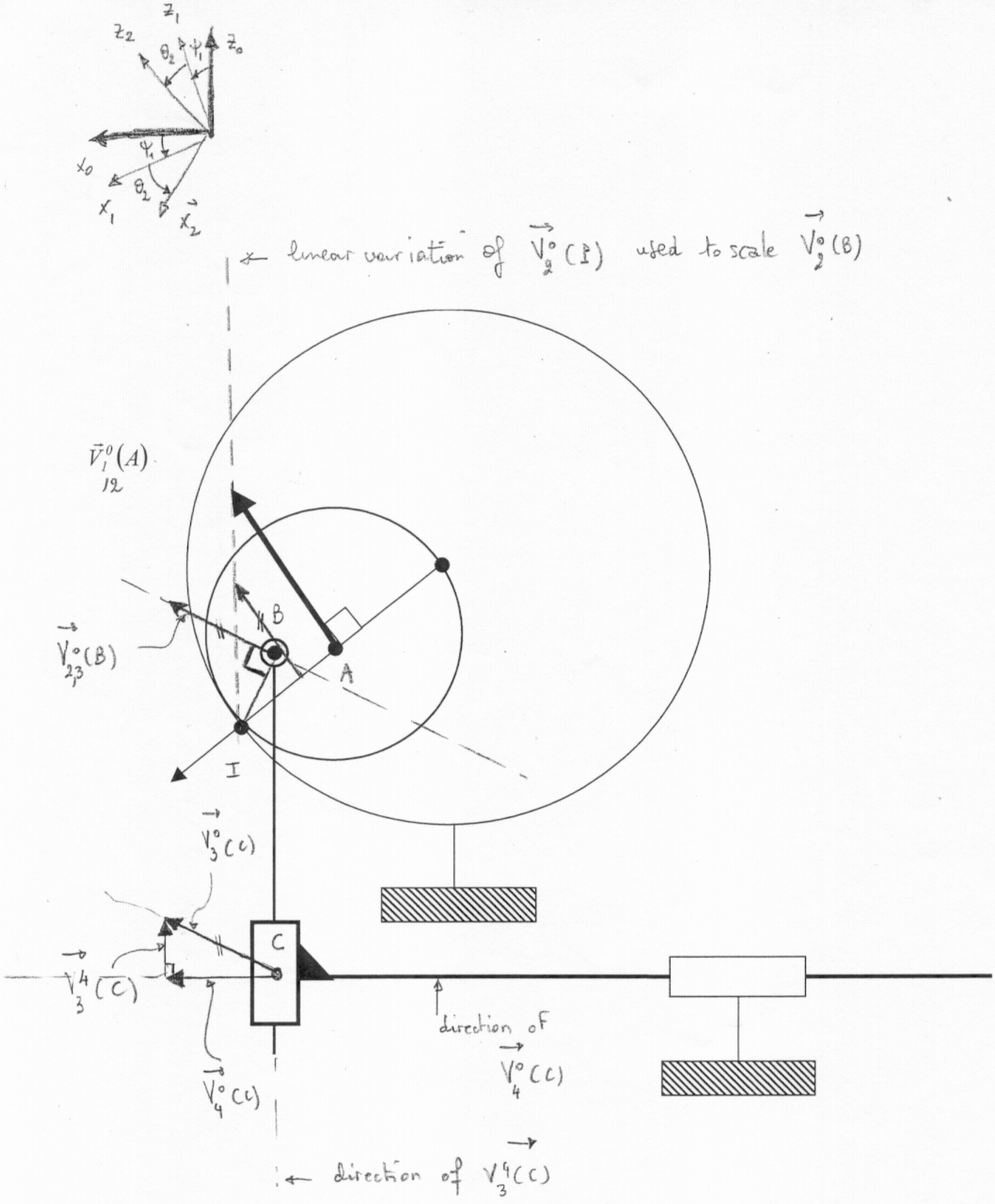
$\vec{V}_1^0(A) = \vec{V}_2^0(A)$

2/0 tangent to a rotation of axis $(I, \vec{y})$ → $\vec{V}_{2,3}^0(B)$

3/0 is a translation $\quad \vec{V}_3^0(C) = \vec{V}_3^0(B)$

3/4 : $\vec{V}_3^4(C) = \vec{V}_3^0(C) - \vec{V}_4^0(C)$ → triangle → $\vec{V}_4^0$(C

direction $\parallel \vec{x}_{3,4}$; known ; ↳ direction $\parallel \vec{z}_{0,4}$



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