PC Deuxième Année
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Physique : Interrogation n° 4

Jeudi 13 mars 2008 **Durée : 1h30**

Barème indicatif : Cours : 5 pts, Exercice 1 : 8 pts, Exercice 2 : 7 pts

Lecture questions

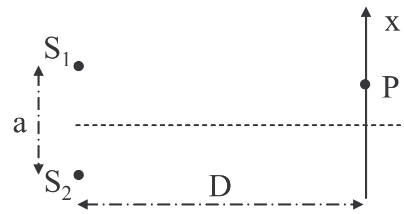
1. One considers a perfect uncharged, medium of permittivity ε, of magnetic permeability μ and of conductivity γ. Using Maxwell's equations which one shall recall, show that the equation of propagation of an electromagnetic wave in such a medium writes :

$$\Delta\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} - \mu\gamma\frac{\partial\vec{E}}{\partial t} = \vec{0}$$

One recalls that : $\vec{rot}(\vec{rot}\vec{E}) = \vec{grad}(div\vec{E}) - \Delta\vec{E}$

2. Interferences with two sources

One considers the interaction of two harmonic acoustic waves emitted by two quasi-ponctual piezoelectric emitters.



a) Under which conditions (about the sources and the receiver), can these two waves produce interferences detectable by a ponctual receiver ?

b) When these conditions are realized, which are the quantities associated to each of the two waves which at any point of space for constructive interferences add : the amplitude, the square of the amplitude, the intensity or the square of the intensity ?

c) One considers a plane perpendicular to the axis of the sources. At a point P of that plane, close to the axis of the sources, the path difference between the two waves can be approximated by $r_2 - r_1 = ax/D$ where a is the distance between the two sources, x the abscisse of the observation point and D the distance between the sources and the observation plane. What is the shape of the interference figure? Justify your answer. Determine the expression of the interfringe as a function of the wavelength λ.

Exercice 1 : Communications with a submarine

The horizontal plane (xOy) separates air, assimilated to vacuum, and the sea, assimilated to a noncharged medium, slightly conductive of conductivity γ, of permeability μ_0 and of relative dielectric permittivity ε_r. One considers an electromagnetic wave propagating in air as a plane harmonic wave (of pulsation ω) and arriving on the interface with the water under a given angle of incidence θ. In the system of axes (0, x, y, z), its angular wave vector $\vec{k}$ has the components (0, k_y, k_z), k_y and k_z being positive constants. The electric field $\vec{E}$ associated with this incident wave writes : $\underline{\vec{E}} = \underline{\vec{E}}_0 e^{j(\omega t - k_y y - k_z z)}$, $\underline{E}_0$ being the complex amplitude of the wave.

1. Make a figure of the considered problem representing the incident wave propagating from the sky to the sea, indicating clearly the axes and their orientation. Express k_y and k_z as a function of ω, θ and c, c being the velocity of propagation of the electromagnetic waves in vacuum.

2. Reflection – transmission at the air-sea interface

One writes the electric fields $\vec{E}_r$ and $\vec{E}'$ respectively associated to the reflected and transmitted waves in the form : $\underline{\vec{E}_r} = \underline{\vec{E}}_{0r} e^{j(\omega_r t - k_{ry} y - k_{rz} z)}$ and $\underline{\vec{E}}' = \underline{\vec{E}}_0' e^{j(\omega' t - k' y - k'' z)}$.

a) Using the propagation equation given in lecture question 1, which holds for any perfect uncharged medium, establish a first relation between k_y, k_z and ω and then a second relation between k', k'' and ω'.

b) Using the boundary condition verified by an electric field at the air-sea interface at z = 0, give the relation between ω, ω_r and ω' and then that between k_y, k_{yr} and k'.

c) Then show that $k''^2 = \dfrac{\omega^2}{c^2}\left(\varepsilon_r - \sin^2\theta\right) - j\mu_0\gamma\omega$.

d) What is the value of k_{rz} ? What is to be deduced from it concerning the angle of reflection as to the angle of incidence ?

3. Now we analyze the conditions of submarine communications between an airplane and a submarine . The frequency band used for such communications varies between 30 Hz and 30 kHz. We shall estimate the depth of penetration of the wave in the sea for that frequency range.
One considers an incident wave arriving on the surface air-sea under a normal incidence $\theta = 0$.

a) Show that, for the considered frequency range, k''^2 can be approximated by : $k''^2 = -j\mu_0\gamma\omega$. (One recalls that $\mu_0 = 4\pi\times10^{-7}$ F/m and one shall take $\gamma = 3\ \Omega^{-1}\text{m}^{-1}$ and $\varepsilon_r = 80$). Deduce from this the two possible values of k''.

b) Show that there then exist two possible solutions for $\vec{E}'$ of the form : $\vec{E}' = \underline{\vec{E}}_0' e^{\pm\frac{z}{\delta}} e^{j(\omega t \pm \frac{z}{\delta})}$, where δ is a positive real. Explicit the coefficient δ, called penetration depth of the transmitted wave, as a function of the data of the problem. Which is the solution $\vec{E}'$ corresponding to a transmitted wave propagating in the direction of increasing z? Justify your answer. How does the amplitude of the transmitted wave vary with the depth z. Justify then the denomination of δ.

c) Determine the numerical value of δ for $f = 30$ Hz and f = 30 kHz ? Comments

Exercice 2 : Vibration modes in wind instruments

The aim of this exercise is to understand how different pitches of notes can be obtained by changing the shape of wind instruments. In the following, ρ is the air density and V the sound speed in air. The air specific impedance, Z, is given by $Z = \rho V$.

1. The transverse flute can be modelled by a cylindrical pipe of length L (figure 1), opened at both ends, one end being the mouthpiece (at x=0) where the instrumentalist excites the air column. The flute has a series of holes judiciously spaced along a generatrix of the cylinder. The instrumentalist has the possibility of opening or closing one or several holes according to the tunes to be played.

a) Determine the frequencies f_n of the vibration modes of a flute of length L.
b) If all the holes of the flute are closed, compute the length L of the flute so that the first vibration mode is a la3 (frequency $f_1 = 440$ Hz).
c) With the same flute, how is it possible to obtain the note which frequency is $f_4 = 4f_1$?

2. As to the clarinet, the body of the instrument is the same as that of a flute but the excitation is made with a reed (vibrating membrane at the level of the mouthpiece). That end (at $x = 0$) is then considered in the clarinet model as being a rigidly closed end, the other end being always open.

a) Write the new boundary conditions and determine the vibrating modes f'_n for a clarinet of length L?
b) Calculate the lowest allowed vibration frequency f'_1, for a pipe of length L (value found in question 1c). Conclusion ? [one recalls that to go an octave higher, the frequency has to be doubled]
c) Explain why a clarinet of length L does not have the same tune as a flute of length 2L.

3. The oboe shape is not a cylinder but roughly a cone. It can thus be modelled by a cone (see figure 2).

a) M being a point inside the cone and r the distance from M to the cone apex O (OM = r), it can be shown that the total pressure takes the following form : $p_{tot} = A \cdot \frac{\sin(kr)}{r} \cdot \cos(\omega t)$.
Comment this expression.
b) Determine the frequencies of the allowed vibration modes f''_n for an oboe of length L. Conclusion ?

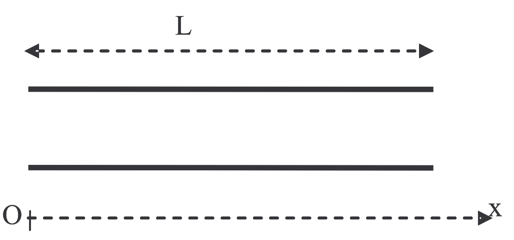


Figure 1 : transverse flute : tube open at both ends

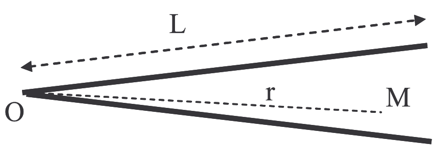


Figure 2 : Oboe

CORRIGE-BAREME
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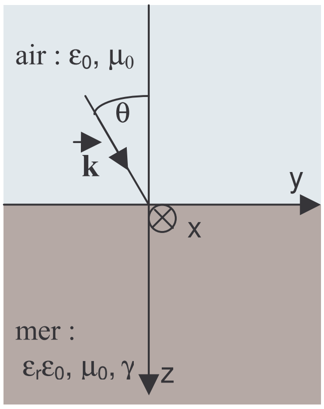
	Questions de cours (5 points)	
1.	Equations de Maxwell : $div\vec{D} = \rho = 0 \Rightarrow div\vec{E} = 0 \quad div\vec{B} = 0 \quad rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $rot\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \Rightarrow rot\vec{B} = \mu(\gamma\vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t})$ $rot(rot\vec{E}) = -\frac{\partial}{\partial t}(rot\vec{B}) = -\mu\gamma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ avec $rot(rot\vec{E}) = grad(div\vec{E}) - \Delta\vec{E}$ on obtient : $\Delta\vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\gamma \frac{\partial \vec{E}}{\partial t} = \vec{0}$	
2.a) **b)** **c)**	- Il faut que les deux ondes aient même pulsation - il faut que les sources soient cohérentes c'est-à-dire émettent avec un déphasage constant dans le temps - Il faut un détecteur quadratique dont le temps de réponse est très supérieur à la période temporelle des ondes. En un point en interférences constructives, seules les *amplitudes* reçues s'additionnent Les lieux d'égale d'interférence correspondent à $\delta = c^{ste} \; soit \; x = c^{ste}$. La figure d'interférences est donc constituée de franges rectilignes d'équation $x = c^{ste}$. $\Delta\delta = \lambda = \frac{a}{D}\Delta x = \frac{a}{D} i \Rightarrow i = \frac{\lambda D}{a}$	
	TOTAL Question de cours	**5 points**

Exercice 1 (8 points)	
1. schéma avec axes conformes au texte  $\vec{k}$ n'a pas de composante suivant x, le plan d'incidence est (yOz). $\underline{\vec{E}} = \underline{\vec{E}}_0 e^{j(\omega t - \vec{k}.\vec{r})}$ avec $\vec{r} = \vec{OM}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ et $\vec{k}\left\lbrace\begin{array}{l} 0 \\ k_y = \frac{\omega}{c}\sin\theta \\ k_z = \frac{\omega}{c}\cos\theta \end{array}\right.$	

2. a)

Milieu 1 : Air : $\Delta\vec{E} - \mu_0\varepsilon_0 \dfrac{\partial^2 \vec{\underline{E}}}{\partial t^2} = \vec{0}$ et $\underline{\vec{E}} = \underline{\vec{E}}_0 e^{j(\omega t - k_y y - k_z z)}$

Soit : $k_y^2 + k_z^2 = \mu_0\, \varepsilon_0 \omega^2$ (1)

Milieu 2 : Mer : $\Delta\vec{E} - \mu_0\varepsilon_0\varepsilon_r \dfrac{\partial^2 \vec{\underline{E}}}{\partial t^2} - \mu_0\gamma \dfrac{\partial \vec{\underline{E}}}{\partial t} = \vec{0}$ et $\underline{\vec{E}}' = \underline{\vec{E}}'_0 e^{j(\omega' t - k' y - \lambda' z)}$

Soit : $k'^2 + \lambda'^2 = \mu_0\, \varepsilon_0\, \varepsilon_r \omega'^2 - j\mu_0\, \gamma\omega'$ (2)

b) La continuité de la composante tangentielle du champ en z = 0 s'écrit :

$$\vec{a}_i \underline{E}_0 e^{j(\omega t - k_y y)} + \vec{a}_r \underline{E}_{0r} e^{j(\omega_r t - k_{ry} y)} = \vec{a}' \underline{E}' e^{j(\omega' t - k' y)}$$

Cette relation devant être vérifiée $\forall t$ et $\forall y$, on a :

$\omega_r = \omega' = \omega$ et $k_{ry} = k' = k_y$

c) D'après la relation (2) obtenue en a) :

$$\lambda'^2 = \mu_0\varepsilon_0\varepsilon_r\omega^2 - j\mu_0\gamma\omega - k_y^2$$

Avec $c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$ *et* $k_y = \dfrac{\omega}{c}\sin\theta$

On obtient : $\lambda'^2 = \dfrac{\omega^2}{c^2}(\varepsilon_r - \sin^2\theta) - j\mu_0\gamma\omega$

d) Même norme du vecteur d'onde pour les ondes incidentes et réfléchies puisque propagation dans le même milieu. Comme d'après b), on a $k_{ry} = k_y$, on en déduit que $k_{rz} = -k_z$ (le – à cause de la propagation de l'onde réfléchie dans le sens des z décroissants).

Donc l'angle de réflexion est égal à l'angle d'incidence.

4. a) $\theta = 0 \Rightarrow \lambda'^2 = \dfrac{\omega^2}{c^2}\varepsilon_r - j\mu_0\gamma\omega$

Il s'agit de montrer que dans la gamme de fréquence considérée

$\omega^2\, \varepsilon_r / c^2 \ll \omega\mu_0\gamma$

$\Leftrightarrow$ $\omega = 2\pi f \ll \mu_0\gamma\, c^2/\varepsilon_r$

$\Leftrightarrow$ $f \ll \sim 2\times 10^7$ Hz qui est bien vérifié pour la gamme de fréquence 30Hz-30kHz

On a donc : $\lambda'^2 = -j\mu_0\gamma\omega$ imaginaire pur.

$\lambda' = \pm(\omega\mu_0\gamma)^{1/2} e^{-j\pi/4} = \pm(1-j)\,(\omega\mu_0\gamma/2)^{1/2}$

b) En remplaçant λ' dans l'expression de $\vec{E}'$, avec $k_y = 0$ puisque $\theta = 0$:

$$\vec{E}' = \underline{\vec{E}}'_0 e^{\pm\frac{z}{\delta}} e^{j(\omega t \pm \frac{z}{\delta})}$$

Avec $\delta = \sqrt{\dfrac{2}{\omega\mu_0\gamma}}$

Pour l'onde transmise se propageant dans le sens des z croissants, la solution est :

$$\underline{\vec{E}}' = \underline{\vec{E}}'_0 e^{-\frac{z}{\delta}} e^{j(\omega t - \frac{z}{\delta})}$$

L'amplitude de l'onde $E'_0 e^{-\frac{z}{\delta}}$ décroît exponentiellement avec z.

δ est le coefficient d'amortissement exponentiel d'où son appellation.

c) Pour f = 30 kHz, $\delta = \sqrt{\dfrac{2}{\omega\mu_0\gamma}}$ = 1,67 m

Pour f = 30 Hz, $\delta = \sqrt{\dfrac{2}{\omega\mu_0\gamma}}$ = 5,3 m La profondeur de pénétration augmente lorsque la fréquence diminue.	
TOTAL Exercice 1	**8 points**

	Exercice 2 (7 points)	
1.a) b) c) d)	Flûte $\underline{p_i} = j\rho V\omega U_M e^{j(\omega t-kx)}$ et $\underline{p_r} = -j\rho V\omega U_M e^{j(\omega t+kx)}$ (surpression nulle en x=0) d'où $\underline{p_{tot}} = 2\rho V\omega U_M e^{j\omega t}\sin(kx)$ et $p_{tot} = 2\rho V\omega U_M \sin(kx)\cos(\omega t)$ $p_{tot} = 0$ en $x = L$ d'où $f_n = n\dfrac{V}{2L}$ avec n entier non nul L = 38,6 cm Fréquence f_4 obtenue en mettant un trou à $x = L/4$ (pour faire un nœud de pression à cet endroit).	
2.a) b) c) d)	Clarinette En $x = 0$, le déplacement des particules doit être nul. D'où $\underline{U_r} = U_M e^{j(\omega t+kx)}$ $u_{tot} = 2U_M\cos(kx)\cos(\omega t)$ et $p_{tot} = 2\rho V\omega U_M\cos(kx)\sin(\omega t)$ $p_{tot} = 0$ en $x = L$ donne $f'_p = p\dfrac{V}{4L}$ avec p entier impair f'_1 = 220 Hz. On a la même note que pour la flûte, mais une octave plus grave. Pour la flûte de longueur 2L, on a $f_n = n\dfrac{V}{4L}$, n entier non nul Pour la clarinette de longueur L, on a $f'_p = p\dfrac{V}{4L}$, p entier impair Ainsi par exemple pour un son de fréquence fondamentale $f_1 = f_1' = \dfrac{V}{4L}$, dans le cas de la flûte on aura tous les harmoniques $2f_1$, $3f_1$, $4f_1$, $5f_1$,…tandis que pour la clarinette, on aura seulement les harmoniques impairs $3f_1$, $5f_1$, …. Ainsi le timbre du son obtenu est différent.	
3.a) b)	On est en coordonnées sphériques car onde incidente sphérique L'amplitude de la pression varie en $1/r$ (énergie constante) On vérifie également qu'en $r = 0$, on a un ventre de pression $p_{tot} = 0$ en $r = L$ d'où $f_n'' = n\dfrac{V}{2L}$ avec n entier non nul. On se retrouve dans le cas de la flûte : cône fermé au sommet équivalent à un cylindre ouvert aux 2 extrémités.	
	TOTAL exercice 2	**7 points**